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What should be the focus of emotion regulation in children? A nonlinear dynamic mathematical model of children's peer interaction in groups

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Abstract

This paper questions the assumption that children's social and emotional competence be placed within the developing child, rather than in the interaction of the child with the range of peer social ecologies in which the children might function. This paper presents a new nonstatistical mathematical approach to modeling children's peer social interaction in small groups using nonlinear difference equations in which both an uninfluenced and an influenced regulatory set point of positive minus negative interaction can be separately estimated. Using this model and the estimation procedure, it is possible to estimate what a focal child and the group initially brings to the group interaction and also how these regulatory set points are influenced by the interaction to determine two influenced regulatory set points. Six-person mainstreamed and specialized groups were established involving three types of unacquainted preschool boys: children with and without developmental delays and a language disordered but intellectually normally functioning group, using a methodology that ensured appropriate matching of child and family characteristics. For each 2-week play group, the social interactions of each child were observed during a designated free play period. Handicapped children were observed in either a specialized or mainstreamed setting. The application made of this modeling process in this paper is generating theory to attempt to understand influence processes. Parameters are introduced that reflect uninfluenced target child and group set points, emotional inertia, and influence functions.

There has been a great deal of recent interest in the general construct of "emotion regulation" in children (e.g., Fox, 1994; Garber & Dodge, 1991; Macobby, 1980; Thompson, 1988). Macobby (1980) converged on the ability of children to inhibit inappropriate behavior in her discussion of emotion regulation. Gottman and Katz (1989) used the construct to include the ability to self-soothe, focus attention, and organize one's self for coordinated action in the service of a specific goal (usually externally imposed, such as a teacher's demand that children correctly spell a list of words).

The child's regulatory physiology and parent–child interaction have played a role in recent thinking on emotion regulation abilities in development. Gottman, Katz, and Hooven (1996) showed that Porges' constructs of vagal tone and the ability to suppress vagal tone (e.g., Porges, Doussard–Roosevelt, & Maita, 1994) at age 4 were related to the way and the extent to which parents "emotion coached" their children when they were angry or sad, and that these regulatory physiological variables were, in turn, predictive of the child's

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ability to down regulate negative affect at age 8, focus attention, achieve in mathematics and reading (controlling for IQ at age 4), have competent peer relations, avoid the development of internalizing or externalizing problems, and avoid infectious illness. Wilson and Gottman (1996) explored the role of attentional processes and underlying regulatory physiology in mediating between emotion and cognition, and in producing positive developmental outcomes.

Two-thirds of Fox's (1994) recent edited monograph was devoted to either the physiological or psychological/psychobiological aspects of emotion regulation. A great deal of this interest in underlying physiology lies in exploring ways in which the child's temperament interacts with potential environments to create various pathological child outcomes. For example, Calkins (1994) proposed the theory that the child's autonomic or central nervous system reactivity to frustration, coupled with controlling, intrusive, and coercive parents would result in aggressive children, while the child's autonomic or central nervous system reactivity to novelty, coupled with overprotective and controlling parents would result in fearful, inhibited, and socially withdrawn children. However, Thompson's (1988) developmental work on what he called "emotion dynamics" (the temporal form of the child's emotional responses) demonstrated that, in the Strange Situation, "distress continued to have a strong, direct influence on reunion-episode attachment behavior *independent* of temperament" (p. 393). Hence, some of the child's emotion regulation abilities are likely to be independent of temperament.

Thompson's (1994) extensive definition of emotion regulation also included ecological contexts, specifically the increasing capability of children as they develop to *select* social contexts that maximize positive affect and meet their needs (p. 37). Carstensen's (1991) selectivity theory also called our attention to the tendency of older people to select settings that guarantee a predictable world that is minimally stressful and maximally rewarding. Thus, Thompson considered this developing capability to consciously and actively select environmental contexts as part of the child's developing skill of emotion regulation, and, therefore, although environmental contexts are mentioned these discussions, they are still part of the child's ability to select the appropriate context.

Hence, definitions of emotion regulation have been located entirely within the developing child, and not in the environmental context, nor in their interaction with the developing child. In studies of children's social and emotional competence we have thus implicitly placed the locus of emotion regulatory competence within the child, rather than in the interaction of the child with the peer social ecologies in which the children need to function. As a result of this conceptualization, our concepts of emotion regulation have been formulated independently of the range of social interaction ecologies and both the challenges and the scaffoldings that these different ecologies offer to the developing child for regulating emotions. While we seem to be quite aware of the role that caregivers play in helping to regulate the emotions of the developing infant (e.g., see Calkins, 1994; Cicchetti, Ganiban, & Barnett, 1991), emotion regulation is still located within the developing infant. Moreover, we have not focused on a variety of settings provided in the child's peer world as sources of the development of emotion regulation abilities. However, the selection of these settings may be of central importance in the design of educational environments, particularly for the development of emotion regulation abilities in developmentally delayed or other specialized populations of high risk children.

Furthermore, young children, or developmentally delayed children may not yet be able to *select* these settings, and so it may be incumbent upon researchers to uncover the inherent variability in these settings that affect a child's ability to regulate affect. Cicchetti (1993) pointed out the importance of studying developmental processes in both normally developing and high risk populations for "uncovering pathways to competent adaptations despite exposure to conditions of adversity" (p. 473). Peer social contexts may be one such source of scaffolding that presents hidden pathways to the development of emotion regulation abilities in high risk children.

Because of the inherent emotional architecture of the human brain, and because of the dynamic homeostases in the human body, "emotion regulation" is an appealing general concept for researchers interested in integrating a biological approach with a dynamic social interactional approach (see Cicchetti & Tucker, 1994). In our previous work in attempting these integrations in the area of marital interaction, we have found it useful in terms of prediction of longitudinal outcomes (Gottman, 1994) to work with variables that index the time-locked amount of affectively positive minus negative interaction as the overall transaction within one ecology is observed over time. We also take that approach in this paper.

In this paper we propose a simple theoretical language for both an individual and a *systemic* notion of emotion regulation, one that involves classical set point theory, in which particular behavioral affective set points or homeostatic settings are defended by the interacting system. In the theoretical language we propose, the focal child would be continually attracted to its natural *uninfluenced* set point, except that nonlinear interactions within the interacting system change the child's trajectory so that it is instead drawn to its *influenced* set point. The equations of the system we propose must include the form of these nonlinear influence functions; once the form of these functions is selected, since they are empirically fit, they are a product of the modeling we propose. The uninfluenced set points might be a function of organismic variables, but the influenced set points are a function of the interaction of these variables with characteristics of the social ecology.

Hence, we suggest that the parameters of emotion regulation we will delineate be considered both a quality of individual temperaments and a quality of a potentially scaffolding peer interacting system, rather than solely a quality of a focal child being observed. The interacting system then provides a qualitative organization of the child's behavior and physiology by determining the set of "affective attractors" in that ecology for that child. This organization, in turn, provides a set of developmental challenges or buffers for the focal child's developing social competence. We suggest that these ecologies can be differentially evaluated in terms of the regulatory mechanism they provide.

We propose this new approach to modeling emotional regulation in children's peer social interaction and illustrate this approach with the mathematics of difference and differential equations. These equations express, in mathematical form, a proposed mechanism of change over time. They do not represent a statistical approach to modeling but rather they are designed to suggest a precise mechanism of change. This method has been employed with great success in the biological sciences (Murray, 1989). It is usually a quantitative approach that requires the modeler to be able to write down, in mathematical form, on the basis of some theory, the causes of change in the dependent variables. For example, in the classic predator–prey problem, one writes down that the rate of change in the population densities is some function of the current densities (e.g., Murray, 1989). While this is a simple representation of the predator–prey phenomenon, it has served well as an initial exploratory model. The equations are designed to write down the precise form of rates of change over time. The ideal mathematical technique for describing change is the area of differential equations. These equations usually used linear terms or linear approximations of nonlinear terms, and they often gave very good results. In fact, most of the statistics we use as a field are based upon linear models. In the area of differential equations, linear equations simply assume that rates of change follow generalized straight line functions of the variables rather than curved line functions.

However, in recent years it has become clear that most systems are complex and must be described by *nonlinear* terms. Interestingly, research workers have discovered that by employing nonlinear terms in the equations of change some very complex processes can be represented with very few parameters. Unfortunately, unlike many linear equations, these nonlinear equations are generally not solvable in closed functional mathematical form. For this reason the methods are called "qualitative," and visual methods must be relied upon. For this purpose, numerical and graphical methods have been developed such as "phase space plots." These visual approaches to mathematical modeling can be very appealing in engaging the intuition of a scientist working in a field that has no mathematically stated theory. If the scientist has an intuitive familiarity with the data of the field, our approach may suggest a way of building theory using mathematics in an initially qualitative manner. The use of these graphical solutions to nonlinear differential equations makes it possible to talk about "qualitative" mathematical modeling. In qualitative mathematical modeling, one searches for solutions that have similarly shaped phase space plots.

These models have been proposed in developmental psychology in the work of E. Thelen (e.g., Thelen, 1995) on the development of infant locomotion, although she did not use equations to build mathematical models. Instead, mathematical constructs of interest to her formed the basis for her more metaphorical discussion of patterns she observed in her data.

We have successfully used this approach in modeling marital interaction (Cook et al., 1995). In that analysis we developed a mathematical model of marital interaction using two difference equations, one to represent the wife's time series and one to represent the husband's time series. The time series were obtained by a suitable weighting of categorical observational codes. Parameters of the model were then subjected to validity tests and to tests of whether they were able to predict divorce or marital stability.

Mathematical Modeling

To illustrate our methods, as our data we used a weighting of codes from the White–Watts ISB coding system of social interactive behaviors during children's peer interaction in small groups. We weighted each code positively or negatively at each block of observation (see Appendix for the weighting scheme); this created a time series for the focal child and a time series for the group. The goal of our mathematical modeling was to conceptu-

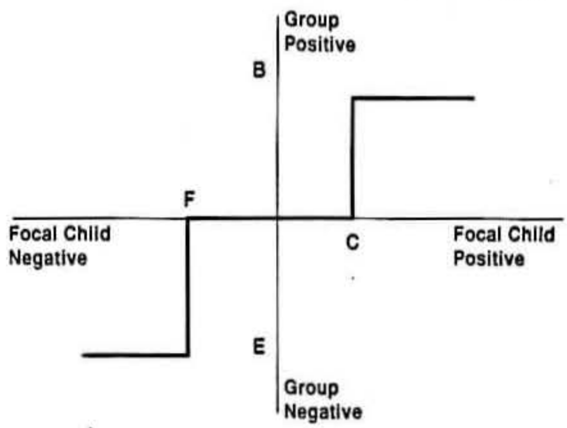


Figure 1. Sigmoidal shape of the influence function.

ally dismantle these two time series into theoretically meaningful parts. We dismantled each of these two time series into two components that represented: a function of interpersonal influence from focal child to group and from group to focal child, and terms containing parameters related to the focal child and the group's own dynamics, independent of influence. This dismantling of scores into "influenced" and "uninfluenced" behavior represents our theory of how the dependent variable may be decomposed into components that suggest a mechanism for peer group interaction, in which the notion of "regulation" is in the ecology rather than in the focal child.

An influence function is used to describe the group's interaction. The mathematical form is represented graphically with the x axis as the range of values of the dependent variable (positive minus negative at an observational block) for either the focal child or the group and the y axis the average value of the dependent variable for the other's immediately following behavior, averaged across all time blocks. We selected as our theoretical form for the influence function the sigmoidal, or S-shaped function, illustrated in Figure 1. In this sigmoidal form positive values are presumed to have a positive influence until they pass a critical threshold, after which this value may change to a more positive value. Negative values are presumed to have a generally negative influence until they pass a critical threshold, after which this value may change to a more negative value. A more reactive focal child (or a more reactive group) has a lower threshold of response. In practice we need not assume such a strict form to the influence function, but instead we can fit the influence function iteratively using the sigmoidal form as a general guide. Thus, it is possible in our model for the influence function to be in any two quadrants of the plane, positive or negative, as long as it has the general shape we describe. The parameters of these influence functions (e.g., the point at which the group's negativity starts having an effect on the focal child) might vary as a function of many other variables, including, we expect, the type of child. For example, we might expect developmentally delayed focal children to be more reactive to negativity from the group than normally developing children.

These influence functions then assume that the amount of influence will remain constant across the remainder of the ranges of the variable, that is, that the influence function will be composed of four flat pieces of varying levels, two levels in the negative range and two levels in the positive range. This is, of course, only one kind of influence function that we could have proposed. For example, we could have proposed that the more negative the dependent variable, the more negative the influence, and the more positive the dependent variable the more positive the influence. We posit this form of the influence function because we have had success with this form in modeling marital interaction, and because there are certain mathematical advantages[1] to an influence function whose slope is flat almost everywhere (zero derivative almost everywhere). We also believe that the threshold parameters of the influence functions will have theoretical meaning.

We begin with a sequence of scores: C_t, G_t, C_{t+1}, G_{t+1}, . . . , where C stands for the focal child, G for the group, and t for turn. We ignore in this process which other child in the six-person group stands in for the group, thus symbolically reducing the interaction from group to dyadic. As in the marital case, for simplicity, we refer to each of the other time series as the "partner's" time series. In the process of modeling, two parameters are obtained for the focal child and two for the group. One parameter is their emotional inertia (positive or negative), which is their tendency of remaining in the same state for a period of time, and their natural uninfluenced set point, which is their average level of positive minus negative when their partner's score was zero, that is, equally positive and negative. For purposes of estimation we assumed that zero scores had no influence on the partner's subsequent score.[2] Having estimated these parameters from a subset of the data, we then subtracted the uninfluenced effects from the entire time series to reveal the influence function, which summarizes the partner's influence. An additional parameter that emerges from our modeling is the influenced set point of the interaction, which is a "steady state set point," or a sequence of two scores (one for each partner) that would be repeated ad infinitum if the theoretical model exactly described the time series; if such a steady state set point is stable, then sequences of scores will ap-

1. This function, the sum of the squares of the derivatives, can be shown mathematically for our influence function in which the derivative is zero almost everywhere to be a Lyapunov function (see Jordan & Smith, 1988, p. 267; Simmons, 1991, p. 465). Lyapunov functions are used to assess stability of steady states, or what we are calling "influenced set points." A mathematical proof of this fact is available from the first author on request.

2. In practice we were able to make this assumption in 93 cases. In 27 of the cases we had no zero points so we assumed that the next lowest value of 1.0 was the point of no influence, and in 3 cases we had no zeros or ones and had to assume that 2.0 was the point of zero influence. We tested the validity of our assumption as follows. We obtained all chains in which the value of one time series stayed 0.0 for the 3 points. If this is a value of zero influence, then the partner's time series should return in exponentially decaying fashion to its uninfluenced set point. We then computed the sum of the squares of deviations from this expected line of return. We made the same computation for the value of 1.0, 2, 0, and so on. The curve should identify the minimum. To accomplish this, we needed to use all the data for all children because there were so few of these 3-point sequences. The data suggested that the curve was low and relatively flat for values of 0.0, 1.0, and 2.0. Hence, this assumption that low values have low influence is justified.

proach the point over time. In a loose sense it represents the average score the theoretical model would predict for each partner. We can then examine whether the influenced set point was more positive than the uninfluenced set point; that is, we can ask did this group ecology pull the focal individual (or the group) in a more positive or a more negative direction, so that the influenced set point is quite different from the uninfluenced? The uninfluenced set point can be thought of as what the individual brings to the group interaction, and the influenced set point as where the ecology then takes the individual. We can then ask our fundamental questions about how differential social ecologies affect the systemic emotional regulation of handicapped and nonhandicapped children.

In addition to these parameters, there are actually six parameters that define the sigmoidal influence function, two thresholds (positive and negative), denoted C and F, respectively, and two levels for positive (A and B) and two levels for negative interaction (D and E) values. We place no restrictions on the sign or value of these parameters, but instead fit them empirically.

Validity test

In addition to our goal of discriminating handicapped from nonhandicapped children using parameters that dynamically describe the group interaction, we will also test the validity of the parameters estimated for each "partner" in each group by correlating these parameters with a set of criterion variables based on another observational system, the Rubin Play Scale, the Achenbach and Edelbrock Child Behavior Checklist, peer sociometrics administered after the 2 weeks of daily group sessions, and two stringent variables we have used in previous research constructed from the interaction data, the number of mutual friends, and the number of children who seek out the focal child. This will give us an idea of how much of the model we need to do work in accounting for variance in criterion variables currently important.

The Model

The model presented in this paper attempts to reproduce the sequence of scores derived from the White–Watts ISB coding system. We have confined ourselves to a deterministic model, regarding any score as being determined by the most recent two scores only. In this way, we use a discrete model to describe the individual's level of positivity in each observational block. That is, we seek to understand interactions as if individual behavior were based purely on predefined reactions to (and interpretations of) recent actions (one's own and one's partner's). This may not be true in the main, but it may be true enough that the results of the model suggest underlying patterns that affect the way any particular group interacts. In the next section the details of the model are and in the following section the methods for estimation of model parameters are described. In the subsequent section the mathematical and the experimental results are presented.

Description of the model

The assumption that each person's score is determined solely by their own and their partner's previous score restricts us to a particular class of mathematical models. If we denote C_t and G_t as the focal child's and the group's scores respectively at turn t, then the sequence of scores is given by an alternating pair of coupled difference equations:

$$C_{t+1} = f(C_t, G_t),$$
$$G_{t+1} = g(C_{t+1}, G_t); \qquad (1)$$

The functions f and g remain to be determined. The asymmetry in the indices is due to the fact that we are assuming, without loss of generality, that the focal child acts first. We therefore label the turns C_1, G_1, C_2, G_2, . . . , and so on. To select a reasonable f and g, we make some simplifying assumptions. First we assume that the past two scores contribute separately and that the effects can be added together. Hence, a person's score is regarded as the sum of two components, one of which depends on their previous score only and the other on the score for their partner's last turn. We term these the "uninfluenced" and the "influenced" components, respectively. Consider the uninfluenced component of behavior first. This is the behavior one would exhibit if not influenced by one's partner. It could primarily be a function of the individual, rather than the group, or it could be a cumulative effect of previous interactions, or both. It seems reasonable to assume that some people would tend to be more negative when left to themselves while others would naturally be more positive in the same situation. This baseline temperament we term the individual's (or the group's) "uninfluenced set point." We suppose that each individual would eventually approach that set point after some time regardless of how happy or how angry they were made by a previous interaction. The simplest way to model the sequence of uninfluenced scores is to assume that uninfluenced behavior can be modeled by a simple linear relationship. This leads us to the linear relationship

$$P_{t+1} = r_i P_t + a_i, \tag{2}$$

where P_t is the score at turn t, r_i determines the rate at which the individual (or group) returns to the uninfluenced set point, and a_i is a constant. The constant r_i will henceforth be referred to as "inertia." As a subject of subsequent research, we expect that the inertia parameters will reflect a subject's own temperament and the subject's chronic level of physiological arousal and inability to self-soothe (with higher physiological arousal implying higher emotional inertia). The uninfluenced set point is the steady state of this equation. It is the point where, when not being influenced, nothing changes over time, so that P_t is constant. This uninfluenced steady state is found by solving $P_{t+1} = P_t = P = a_i/(1 - r_i)$. The behavior of this difference equation is governed by the value of r_i. If the absolute value of r_i is less than 1.0, then the system will tend toward the steady state regardless of the initial conditions, while if the absolute value of r_i is greater than 1.0, the system will always evolve away from a steady state.

Clearly we require the natural state to be stable, and so we are only interested in the case in which the absolute value of r_i is less than 1.0. The magnitude of r_i determines how quickly the uninfluenced state is reached from some other state, or how easily a person (or the group) changes frame of mind, hence the use of the word "inertia." The influence function is a plot of one person's behavior at turn t on the x axis, and the subsequent turn $t + 1$ behavior of the peer on the y axis. Averages are plotted across the whole interaction. Recall that the theoretical influence function that we selected was sigmoidal. In particular, there is evidence that in young children's peer interaction, the processes of influence often give rise to negative affect, but that the effects of this negativity are often prosocial if aggression is controlled (Shantz & Hartup, 1992). Hence, the most important aspect of the sigmoidal influence function is that the influence is zero near zero, and that the function is flat piecewise, except for a few jumps in level. Other theoretical forms of the influence function are possible; for example, we are experimenting with the hyperbolic tangent function.

We denote the influence functions by $I_{AB}(A_t)$, the influence of person A's state at turn t on person B's state. With these assumptions the complete model is

$$C_{t+1} = I_{GC}(G_t) + r_1 C_t + a, \tag{3}$$

$$G_{t+1} = I_{CG}(C_{t+1}) + r_2 G_t + b. \tag{4}$$

Again, the asymmetry in the indices is due to the fact that we are assuming (without loss of generality) that the focal child acts first. The problem now facing us is estimation of our four parameters, r_1, a, r_2, and b, and the empirical determination of the two unknown influence functions.

Estimation of parameters and the unknown influence functions

To isolate and estimate the uninfluenced behavior we look only at pairs of scores for one person for which the intervening score of their partner was zero (about 15% of the data). Because by assumption at these points where $C = 0$, we assume that $I_{CG} = 0$ (and, analogously, at points where $G = 0$, we assume that $I_{GC} = 0$), Equations (3) and (4) collapse to Equation (2) and we can use least squares on these subsets of the data to estimate the two unknown constants for each person. Note that we can now compute the uninfluenced states and inertia of each partner (see footnote 2).

Once we have estimated the uninfluenced component of the scores we can simply subtract it from the scores at turn $t + 1$ to find the observed influenced component. We can, for example, plot the influenced component of the target child's score against the group's previous score. For each value of the focal child's score during the interaction there is likely to be a range of observed values of the influence component due to noise in the data. To convert these into estimates for the influence functions of the model (I_{CG} and I_{GC}), we simply average the observations for each partner score. Both the raw influence data and the averaged influence function can be plotted for each "component" of the group. This latter concept is tricky, and it would be well for the reader to review it several times to understand how the influence functions are computed. Computationally, we used an iterative procedure to estimate the thresholds, C and F, as well as the parameters A, B, D, and E, for each of the two influence functions.

To validate the estimation process we then form a reconstructed interaction from the model equations. We simply start by taking both the focal child and the group to be at their uninfluenced state (noninteger values are allowed in this reconstruction) and then iterate forwards for the approximately 60 turns of interaction each we tend to observe in 1 hr of observation per focal child. This is done by computing the components separately and then summing to generate the next score. The uninfluenced component is derived from the use of Equation (2). The influenced behavior is computed by simply rounding the partner's last score to the nearest integer and reading off the influence from that person's average influence function, referred to above. The reconstructed interaction therefore lacks any randomness; we do not pretend that this "expected" conversation would ever be observed in practice. Rather, it represents an underlying trend.

Simulation capability. We note that an amazing hidden advantage of this modeling process is that we can now simulate the behavior of the group and the focal child under conditions that are different from those under which we first observed them. For example, we can ask what would happen if the focal child's uninfluenced start value were made more positive, and we can attempt to test the simulation with an experiment in which this change is accomplished. As such this model then leads to predictions and experiments, and subsequent improvements in the model, which is one of its intended contributions.

Steady states and stability

For each group we will plot a "phase plane" containing the model's "null clines." These are all concepts from mathematics, and they follow from the equations themselves. Definitions follow. The phase plane refers simply to the plane with the group's and the focal child's scores as coordinates. Hence, a point in this plane is a pair representing the group's and the focal child's scores for what is usually called a particular "interact" (a two-turn unit). As time progresses, this point moves, and charts a trajectory in phase space. In phase space there are the points called "stable steady states, or set points." These are points that the trajectories are drawn toward, and if the system is perturbed away from these states, it will be drawn back. Unstable steady states are the opposite: if perturbed the system will drift away from these points. Hence, it is of considerable importance to find the steady states of the phase plane.

This is accomplished mathematically by plotting the null clines. Null clines involve searching for steady states in the phase plane; they are theoretical curves where things stay the same over time. A person's null cline is a function of their partner's last score and gives the value of their own score when this is unchanged over one iteration, $C(t + 1) = C(t)$. This last equation says that things stay the same over time, and that is precisely how we find the shapes of the null clines. Plotting null clines provides a graphical means of determining steady states. Simple algebra gives the form of these null clines as

$$C(G_t) = (I_{GC}(G_t) + a)/(1 - r_1),$$
$$G(C_{t+1}) = (I_{CG}(C_{t+1}) + b)/(1 - r_2). \qquad (5)$$

Notice that these are simply the influence functions, scaled (by $1 - r_1$, or $1 - r_2$) and translated (by a or b). In other words, the null clines have the same shape as the influence functions, but they are moved over (translated) by a constant, and they are scaled by another constant. Null clines often play an important role in mathematical analysis since they give a visual indication of the dynamics of the system. The influenced set points, or the equilibria or steady states of the interacting system, are completely determined by the intersections of the null clines, because, by definition, if the system started at this point then it would stay there. Of course, the stability of these steady states to perturbations is yet to be determined. Since we have specified the functional form of the influence functions, we can proceed quantitatively.

The pair of equations in Equation (5) can be solved graphically. The method is identical to solving two simultaneous linear equations ($ax + by = c$; $dx + ey = f$). If these two lines are plotted on the same graph, the point where they intersect gives the solution value (x, y) that satisfies both equations. Therefore, if we plot the two curves from Equation (5) their solution will be given by any points where the curves intersect. Call one of the partners A. Under what conditions will two consecutive scores for A be identical? For any particular score for A there is only one intervening score for A's partner that allows this. Thus, the map from A's score to the partner's score that leaves it unaltered defines a function. This is what we have called A's null cline.

A's partner has a null cline that can be computed in a similar way. We plot two examples of the focal child's and the group's null clines against their corresponding axes in Figure 2. The two plots differ in that the intersection of the null clines in one case demonstrates a stable positive steady state and in the other a stable negative steady state. We are plotting two functions: The value of the focal child C_t for which $C_{t+1} = C_t$ for any given intervening G_t, and the converse for the group. Intersection points are, by definition, points for which both the focal child's and the group's score remain constant on consecutive turns. These points we call the "influenced steady states or set points," and are also referred to as the "system attractor." If a group were to reach one of these states during a conversation, they would theoretically remain there with each partner scoring the same on each of their future turns of speech. If they were perturbed away from one of these steady states, they would be drawn back to it. These *potential flow lines* can be used to map potential trajectories, or solutions to the equations in phase space. There may be many influenced steady states (depending on the influence functions and the uninfluenced parameters): in our data 88.9% involve one steady state, 1.6% no steady states, 5.6% involve two steady states, and 4% involve three steady states.

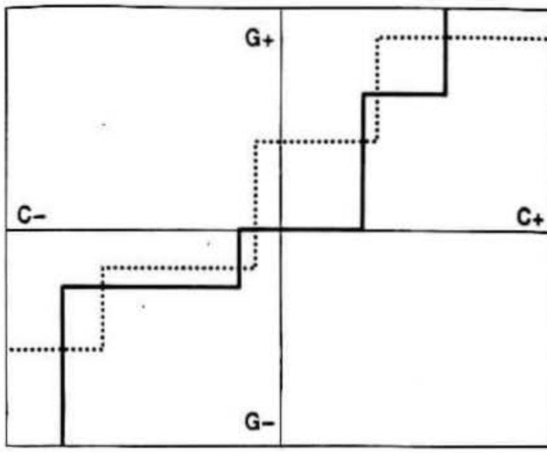


Figure 2. Plot of the null clines showing three stable and two unstable states.

As we noted, there are two types of steady states, stable and unstable. If a theoretical conversation were continued for some time, then pairs of scores would approach a stable steady state and move away from an unstable one. We call the set of points that approach a stable steady state (we ignore the possibility of cycles) the "basin of attraction" for that steady state. An example of a sequence of scores is shown in Figure 2 approaching the more positive steady state. This "theoretical conversation" would be constructed by simply applying Equations (3) and (4) iteratively from some initial pair of scores. The potential existence of multiple stable steady states each with its own basin of attraction has practical implications. The model suggests that the final outcome (positive or negative trend) of a conversation could depend critically on the opening scores of the focal child and the group. Where one begins in the phase space is determined by the group's actual initial conditions. We have generally found that the end points can depend critically on starting values.

An observed or a "reconstructed" interaction can be represented in the phase plane as a series of connected points. In addressing the issue of stability of the steady states, we are asking whether the mathematical equations imply that the reconstructed series will approach a given steady state. Analytically, we ask the question of where a steady state will move once it is slightly perturbed from its position. The theoretical (stable or unstable) behavior of the model in response to perturbations of the steady states is only possible once we assume a functional form for the influence functions. For example, as we have noted, for the sigmoidal influence function, we can have one, three, or five steady states (see Figure 2). From the null-cline plot (see Figure 2) we can see that there are three stable and two unstable states.

What does it mean for there to be multiple steady states? These are all possible states for a particular group. Even if we only observe the group near one of them in our study, all are possible for this couple, given the equations. Each stable steady state will have a "basin of attraction." This is the set of starting points from which a reconstructed time series will approach the steady state in question. If there is a single steady state, then its basin of attraction is the whole plane; that is, no matter what the initial scores were, the sequence would approach this one steady state. We have found this to be the usual situation in our data. If, on the other hand, there are two

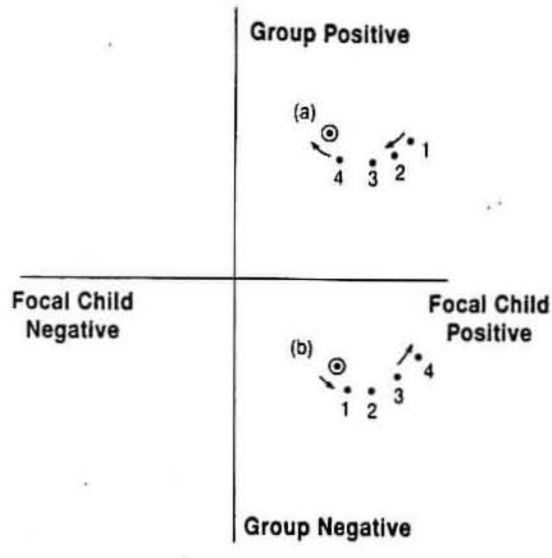


Figure 3. Phase space plots showing (a) a stable steady state, in which perturbations result in return to the steady state and (b) in which perturbations result in movement away from the steady state.

stable steady states (and, necessarily, one unstable one) generally the plane will be divided into two regions (the basins of attraction). If the scores start in the first stable steady state's basin of attraction, then, in time, the sequence of scores will approach that steady state. The same goes for the second steady state and its basin of attraction. This situation is depicted in Figure 3. The group begins at the point (C_1, G_1) in phase space, next moves to the point (C_2, G_2), and the next moves to the point (C_3, G_3), and so on, heading for the large black dot that represents the stable steady state intersection of the two null clines. Notice that this implies that the eventual trend that the conversation follows can be highly dependent on the initial conditions. Thus, high inertia, high influence groups (who are more likely to have multiple steady states) could potentially exhibit a positive conversation on one day and yet not be able to resolve conflict on another. The only difference could be the way the interaction began (their initial scores). The influence functions and uninfluenced parameters would be identical and characteristic of that group. This discussion makes concrete the general systems theory notion of "first-order" (or more superficial, surface structure) change and "second-order" (or more meaningful, deeper structure) change. In our model, first-order change means that the steady states may change but not the influence functions; second-order change would imply a change in the influence functions as well.

Strength of the attractors

As we have noted, the influenced set point for a focal child represents an "attractor" in the following sense. If the system is perturbed from that attractor, it will move back toward it, if it is a stable set point (or "steady state" in mathematical lexicon). The attractor can be positive or negative, and there can be more than one attractor. We can plot the location of the set point in phase space. The phase space is a plot of the values over time taken by the focal child and the group. We can also plot in a third dimension the strength of the attractor, which is the sum of the squares of the derivatives (see Appendix B). (The time derivative for C is simply $C_{t+1} - C_t$, and the time derivative for G is simply $G_{t+1} - G_t$.) Our rationale is as follows. One effect of an intervention might be to strengthen the positive attractor and weaken the negative attractor for a group. Interactions would then have a greater tendency to drift toward the positive and away from the negative attractor. Figure 4 is an example of a 3-dimensional plot for two children's interaction with his group. Flow lines in $G \times C$ phase space, which are the rate of change of G with respect to C, are estimated as the ratio of the two time derivatives, when this ratio exists. In one plot we see that there is a taller mountain peak in the negative–negative quadrant than in the other plot. This taller peak means that there is a greater likelihood that interactions for this child in this group are more buffered from interactions that are highly negative than is the case for the other child interacting with his group.

Methods

Overview

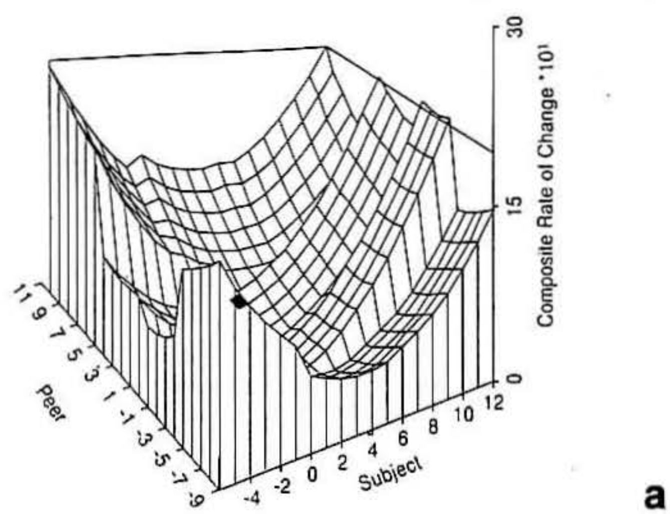
Previously unacquainted groups of children were brought together to form a series of 20 separate play groups ($n = 6$ boys per play group). The play groups differed in terms of two factors: (a) the developmental status of the children in the play groups (referred to as the group variable), that is, children with developmental (cognitive) delays, children with communication disorders, or normally developing children; and (b) the social environment (referred to as the setting variable), that is, play groups consisting only of other children with similar developmental characteristics (all normally developing children or all children with developmental delays) or those in which children from both groups participated. No group contained children with both types of disabilities.

Of the 21 play groups 9 were specialized: 3 consisting of only normally developing children, 3 consisting of only children with communication disorders, and 3 consisting of only children with developmental delays. The remaining 12 play groups were mainstreamed, each consisting of four normally developing children and two children with either developmental delays or communication disorders. Each child participated in only one play group.

As described below, a matching procedure ensured that normally developing children assigned to mainstreamed or specialized play groups, as well as children with communication disorders or developmental delays assigned to mainstreamed or specialized play groups, were equivalent within each of the two types of play groups (settings) in terms of child characteristic measures (chronological age, cognitive, language, adaptive behavior, and behavior problems). A similar matching process ensured equivalence across *all* groups (developmental status and setting) for family demographic measures (family social status, marital status). For each 2-week play group, the social and play interactions of each child were recorded during a designated free-play period. At the conclusion of each play group, peer sociometric ratings were completed for each of the six children.

Participants

Normally developing children were recruited through direct contact with administrators and teachers of public and private nursery schools



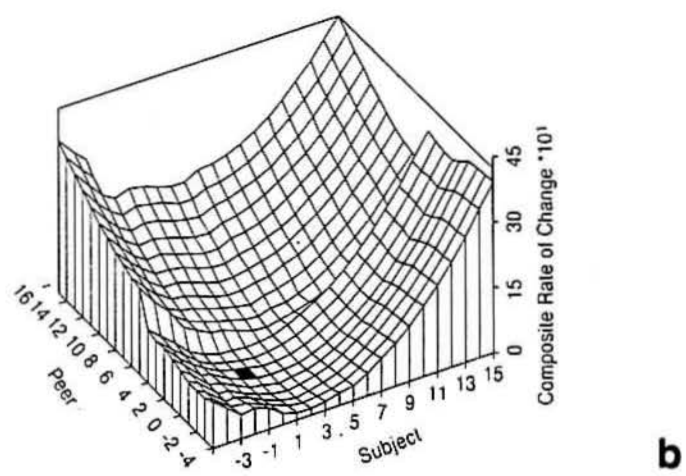


Figure 4. Landscape plot of the Lyapunov function that is the sum of the squares of the time derivatives. Stable steady state is darkened. (a) Note high peak in negative–negative quadrant; (b) note low peak in negative–negative quadrant.

and daycare programs. Children with communication disorders or developmental delays were recruited from community-based preschool programs and from rosters of children who received clinical evaluations from diagnostic clinics. The chronological age range for all subjects was established at 4 years 3 months to 5 years 6 months. Only boys were selected to participate in the play groups because resources ere not available to include gender as an additional independent variable, and more boys were available in community preschools. Similarly, to avoid potential confounds due to race, only Caucasian children were selected. In addition, children were excluded from participating for any of the following reasons: three siblings within 3 years-of-age of the child being considered, teacher reports of major disruptive behavior problems, legally blind or major uncorrected hearing loss, significant motor problems, acquainted with other children in the play group, and living with the primary caregiver less then 1 year.

For selection and matching purposes all prospective children were administered individually the revised version of the Wechsler Preschool and Primary Scale of Intelligence (WPPSI-R; Wechsler, 1989). Full Scale IQ (FSIQ) scores as well as performance (PIQ) and verbal (VIQ) scores were obtained. Two language tests also were administered individually to each child. First, the revised version of the Test for Auditory Comprehension of Language (TACL-R; Carrow–Woolfolk, 1985) was administered. The TACL-R consists of scales for word classes and relations, grammatical morphemes, and elaborated sentences. A total score (standard score) is also obtained. Second, to supplement the receptive language assessment of the TACL-R, the expressive components of the Preschool Language Scale were administered (Zimmerman, Steiner, & Pond, 1979). Because of the lack of standardization, only raw scores were used (range 0–48 for verbal ability and 0–23 for articulation).

In addition to cognitive and language measures, mothers served as respondents for assessments of their child's adaptive behavior and behavior problems. First, the Vineland Adaptive Behavior Scales (Sparrow, Balla, & Cicchetti, 1984) Survey Form was administered to mothers by trained interviewers. Standard scores were obtained for each of the four domains (communication, daily living skills, socialization, and motor skills), as well as for the total adaptive behavior score. Second, the mother's assessment of her child's behavior problems was obtained from the Child Behavior Checklist (CBCL; Achenbach & Edelbrock, 1981). Mothers rated the frequency of different behavior problems from a 118 item questionnaire using a 3-point scale. Only the broad band internalizing and externalizing scales (T scores) in conjunction with a total behavior problem score were used for subject selection and matching purposes. Higher scores indicate greater perceived behavior problems. Finally, responses to a parent questionnaire provided basic demographic information. The Hollingshead Four Factor Index of Social Status (Hollingshead, 1975) was used to calculate a measure of family status (range 8–66).

Beyond the inclusionary and exclusionary criteria applied to all subjects noted above, specific criteria also were established for each of the three groups of children differing in developmental characteristics. Specifically, normally developing children were included if they achieved a FSIQ score between 90 and 130. Children were excluded, however, for any of the following reasons: (a) VIQ or PIQ lower than 90, (b) TACL-R total score less than 90, (c) CBCL Total Problem score greater than 90 percentile, (d) enrolled in a preschool program in which more than 15% of the children have established disabilities, or (e) has a sibling with an established disability.

For children with communication disorders, the selection criteria were more complex. To be included a child must have achieved a PIQ equal to or greater than 90 or a FSIQ greater than 85, and have completed a comprehensive speech, language, and hearing assessment administered by qualified personnel resulting in a categorical diagnosis of a communication disorder and a recommendation for regular therapy. In addition, as a minimal protection against possible diagnostic errors, particularly in view of the wide variability in testing procedures found in the community, and to more carefully define the study population to be included in the sample, children were required to meet one or more of the following criteria: (a) a PIQ > VIQ differential on the WPPSI-R of at least 15 points, (b) a TACL-R total score equal to or less than 90, or (c) a diagnosed articulation disorder. Children were excluded if they obtained a TACL-R score less than 55, a CBCL Total Problem score greater than the 98th percentile, held a primary diagnosis of stuttering, or had an unrepaired cleft palate. The criteria were met by the 30 children with communication disorders who participated in the specialized and mainstreamed play groups as follows: (a) articulation disorder only ($n = 6$); (b) PIQ-PVQ differential greater than or equal to 15 points only ($n = 11$); (c) TACL-R equal to or less than 90 only ($n = 4$); and children with both a PIQ-PVQ differential and low TACL-R score ($n = 4$). In addition, comparisons among children with high and low receptive language scores (based on a TACL-R score of 90 as the cutoff point) and high and low expressive language scores (based on PLS, median split with a cutoff score of 25) revealed an even distribution of children with receptive only, expressive only, and both receptive and expressive deficits. The rationale for this complex series of criteria for selecting children with communication disorders reflects the general lack of agreement in the field for identifying research subjects (McCauley & Demetras, 1990), and the fact that it was not possible nor appropriate for the research staff to administer a complete diagnostic battery. As noted, all children were required to have received a categorical diagnosis of a communication disorder and be recommended for regular therapy by qualified specialists. With regard to our additional criteria, it is important to point out that the TACL (nor revised) is the most frequently used standardized test for selecting children with language impairments (McCauley & Demetras, 1990). Moreover, although it is recognized that PIQ > VIQ differential is not sufficient to guarantee the existence of a communication disorder, it is nevertheless characteristic of children so diagnosed (Stark & Tallal, 1981).

Similarly, children with developmental delays were included if they achieved a FSIQ between 52 and 80. Children in this group were excluded, however, for any of the following reasons: (a) PIQ greater than 90, (b) CBCL total problem score greater than the 98th percentile or teacher reports of continuous and substantial disruption, and (c) a TACL-R total score less than 55 or greater than 90.

Matching procedures

Children with communication disorders or developmental delays were first identified for each play group, with normally developing children participating in both mainstreamed and specialized groups subsequently recruited from the same neighborhoods to maximize similar demographic characteristics. Children were tested on a continuous basis across a 4-year period, and play groups were formed when an appropriate number of children meeting criteria were recruited. Specialized and mainstreamed play groups were interspersed over the 4 years. On occasion, a child meeting established criteria was not included if his test scores were inconsistent with matching projections for the demographic and child characteristic measures.

As a result of this process, child characteristic measures were equivalent for the normally developing children participating in the mainstreamed and specialized play groups ($p > .05$). (The only exception was that normally developing children participating in specialized play groups had a higher Vineland Daily Living Skills score than normally developing children participating in the mainstreamed play groups [$p < 0.5$].) Although 126 children were recruited and participated in the 21 play groups, five subjects were excluded from the mutual friendship analyses (three children with developmental delays and two with communication disorders, one each in specialized and mainstreamed groups) because of failure to meet a minimum criterion of 12 social interactions. Equivalent scores also were obtained across all child characteristic measures for children with developmental delays participating in specialized and mainstreamed settings. In addition, to further ensure an appropriate match between children with communication disorders participating in specialized and mainstreamed settings, similar proportions were maintained for children selected on the basis of PIQ > VIQ differential, the low TACL-R Full Scale score (<90), and a diagnosis of articulation disorder. As noted, only a small number of children received a diagnosis of articulation disorder only.

As expected, significant differences were obtained for most of the child characteristic measures when comparing normally developing children, children with communication disorders, and children with developmental delays. The only exceptions were child's chronological age, the PIQ–VIQ discrepancy, and the CBCL externalizing factor ($p > .05$). Finally, for family demographics, 88.4% of the mothers were partnered, with an average family social strata based on the Hollingshead index of 45.91 (medium business, minor professional). The six groups did not differ for these two measures (χ^2 and Kruskall–Wallace 1-way ANOVA, respectively, $p > .05$). Finally, because normally developing children participated in mainstreamed play groups containing either children with communication disorders or developmental delays, it was important to confirm that the two groups of normally developing children were equivalent. Separate ANOVAS comparing normally developing children in the two types of mainstreamed groups ($n = 24$ per group) revealed no differences on family characteristics or child characteristics, with only one exception found for the daily living skills scale of the Vineland Adaptive Behavior Scale, F (1, 40) $= 5.01$, $p < .05$. In view of the large number of ANOVAS carried out and the fact that neither the Vineland Total Score nor the three other scales yielded significant differences, this daily living scale finding was not given further consideration in the analyses.

Play group setting and procedure

Each six-child play group operated 2.5 hr per day, 5 days per week, for 2 weeks (10 sessions) in either a morning or afternoon time period. Children arrived in separate vehicles (via parents or drivers), and parents were asked to avoid contact with the other families or children for the duration of the play group. Parents were paid $100 plus transportation expenses.

Play groups were supervised by a teacher and graduate assistant in a specially designed laboratory playroom. Children participated in a series of group and individual activities typical of preschool programs, including circle time, music, art, snack, and story. During two daily 30-min free-play periods, children had access to the extensive array of toys and equipment found in the playroom. Separate areas provided opportunities for housekeeping, blocks, puzzles, games, and precast and manipulative toy play activities, as well as an option for individual reading. Although the teacher generally encouraged social and play interactions among the children in other activities, during free-play periods the teacher limited her interactions to providing assistance when necessary.

Using split-screen technology, children's social and play interactions were videotaped by two remote controlled cameras mounted at either end of the playroom and a hand-operated camera in an adjacent observation room. The child being recorded at the time (focal child) wore a specially designed lightweight vest equipped with a professional quality wireless microphone and transmitter secured in a hidden pocket in the back of the vest. Other microphones were placed discreetly throughout the room and a control panel of mixers balanced the auditory signals.

Each child was observed for a total of 60 min during free play over the 2-week period. Recording commenced on the second play group day and was divided into segments of 10 consecutive minutes for each of six recording periods per child. The order of recording children was randomized within blocks of six 10-min segments, and no child was observed more than once per day. In addition, recordings were distributed such that each child was videotaped on three occasions within the first week (Time 1) and on three occasions during the second week (Time 2).

As described below, videotaped recordings were analyzed using two separate scales: one focusing on more global measures of social participation and cognitive play, and the other on individual social behaviors. At the completion of the study, a peer sociometric measure was administered to each child.

Observational measures

Social participation and cognitive play. Parten's (1932) index of social participation formed the basis for characterizing global differences in children's peer relationships. Despite legitimate concerns regarding the sequential and hierarchical nature of this measure of social participation (Bakeman & Brownlee, 1980; Roper & Hinde, 1978; Rubin, Maioni, & Hornung, 1976; Smith, 1978), variations and modifications of the Parten scale, many including measures of cognitive play based on Smilansky's (1968) categories (see Rubin, 1985), appear to have considerable utility. Various forms of the scale have been shown to be sensitive to developmental changes over time (Barnes, 1971; Rubin & Krasnor, 1980; Rubin, Watson, & Jambor, 1978; Smith, 1978), to socioeconomic status (Rubin et al., 1976), to environmental conditions (Vandenberg, 1981), to the familiarity of peers (Doyle, Connolly, & Rivest, 1980), and to differences between mixed-age and same-age groupings (Goldman, 1981). Moreover, variations of the scale have been applied effectively to populations of children with disabilities (Guralnick & Groom, 1985, 1987; Guralnick et al., 1995, 1996a, 1996b; Higgenbotham & Baker, 1981), and may well be of value in identifying children at risk for developmental problems (Rubin, 1982; Rubin, LeMare, & Lollis, 1990).

A time code superimposed on each videotape in conjunction with a remotely controlled tape-stop device allowed observers to view tapes at 10-s intervals. Coders recorded the categories of social participation and level of cognitive play (where required) during each 10-s interval using a slightly modified version of the scale developed by Rubin (1985). This scale consists of 10 mutually exclusive and exhaustive categories. The first three were derived from Parten's (1932) social participation categories consisting of the following play classifications: (a) solitary (playing alone), (b) parallel (playing next to another child), and (c) group (playing with another child; a combination of Parten' associative and cooperative play categories). Nested within these three social participation categories are four measures of cognitive play based on the work of Smilansky (1968): (a) functional (simple repetitive play), (b) constructive (learns to use materials, creates something), (c) dramatic (role taking and pretend play), and (d) games with rules (child behaves in accordance with prearranged rules). If any 10-s interval was coded as either solitary, parallel, or group play, then one of the four cognitive play categories was also scored.

The seven remaining categories consisted of the following: (a) unoccupied behavior (child not playing), (b) onlooker behavior (child watches other children but does not enter into play), (c) reading or listening (reading, leafing through a book, listening to a tape), (d) exploration (examining physical properties of objects), (e) active conversation (talking, questioning, and suggesting to other children but not playing), (f) transition (moving from one activity to another), and (g) adult directed (any activity with an adult).

In order to obtain information with regard to whom the focal child interacted with, the identity of the peer for the group, parallel play, active conversation, and onlooker categories was noted whenever these categories were coded. When more than one child was involved in the interaction, the one in closest proximity to the focal child was coded. More specific definitions for the social participation and cognitive play categories can be found in Rubin's (1985) manual. Coding rules and related modifications of this scale as well as the coding manual for the Individual Social Behavior Scale (see below) may be obtained by writing the first author.

Individual social behaviors. Each videotape was reviewed a second time in order to examine specific peer-related social behaviors. For this purpose, the Individual Social Behavior Scale was developed based on the work of White and Watts (1973) and adapted in a manner similar to Doyle et al. (1980) and to Guralnick and Groom (1985, 1987). The current adaptation was most recently applied by Guralnick et al. (1996a, 1996b) to children with communication disorders. The cluster of individual social behaviors originally described by White and Watts (1973), including the ability to gain the attention of peers, to use peers as resources, to express affection, and to direct peers successfully during play, has been employed extensively. These component behaviors increase over the preschool years, correspond to other measures of social competence with peers such as teacher ratings and peer sociometrics, vary with the familiarity of interacting children, and correlate positively with social participation (Connolly & Doyle, 1981; Doyle et al., 1980; Wright, 1980).

Specifically, observers recorded continuously the occurrence of individual social behaviors defined by 34 categories. The following categories were designed to record social interactions of the focal child as directed to peers: (1) seeks attention of peer, (2) uses peer as a resource, (3) leads in peer activities—direct, positive or neutral, (4) leads in peer activities—indirect, positive or neutral, (5) leads in peer activities—direct, negative,

(6) leads in peer activities—indirect, negative, (7) imitates a peer, (8) involved observation of peer, (9) joins peer(s) in specific activity, (10) verbally supports peer's statement, (11) verbally competes with peer, (12) shows pride in product to peer, (13) competes with peer for adult's attention, (14) expresses affection to peer, (15) shows empathy toward peer, (16) expresses hostility toward peer, (17) takes unoffered object, (18) defends property, and (19) seeks agreement from peer.

With the exception of the involved observation and defends property categories, each of the focal child individual social behaviors listed above was classified as to whether it was an initiation. A focal child initiated event is one in which no prior verbal or nonverbal interaction occurred for at least 3 s.

Fourteen of the remaining categories focused on the social behaviors of the focal child in response to directed activities of peers. Categories consisted of following the lead of a peer (four categories tied to direct/indirect and positive, neutral/negative dimensions), failing to follow the lead of a peer (four categories as above), responding and failing to respond to a peer's attempt to use the focal child as a resource (two categories), responding and failing to respond to peer's attention seeking behavior (two categories), and responding and failing to respond when a peer sought agreement from the focal child (two categories). The final category was one in which the focal child served as a model for a peer.

Ten of the categories designed to record the social interactions of the focal child as directed to peers (1–6, 13, 17–19) also were judged as either successful or unsuccessful. Definitions for successful or unsuccessful social interactions were specific to each social behavior category. For example, the gains the attention of peer category would be coded as successful if the peer attended with 5 s, either visually or verbally, or moved closer to or touched the focal child. The response of the peer must be appropriate to the attention-getting effort of the focal child. Finally, the identity of the peer interacted with also was recorded following procedures outlined above for the social participation scale.

Coders were free to review any segment of the tape as often as needed. The coding protocol was divided into 30-s intervals following the time codes superimposed on the tape. Although coding was continuous, these divisions provided a structure for the coding task and served as a framework for establishing reliability (see below) within the event-based system.

Weighting scheme. Using this observational coding system, a weighting scheme was applied to each of the categories and summed over each observation so that a score was obtained for the focal child and the other child with whom the focal child was interacting. In this way two time series were created, one for the focal child and another time series for "other." Since the identity of the other child varied across the observations, this information was ignored, and the "other" time series represented the group. In this way, for purposes of dynamic modeling, the group interaction was reduced to a generalized dyadic interaction. See Appendix A for details on the weighting scheme.

Peer sociometric ratings. Following Asher, Singleton, Tinsley, and Hymel (1979), at the end of the play group each child was individually presented with color Polaroid photographs of each play group participant and asked to place the photographs into one of three boxes. One box contained a drawing of a happy face for "children you really like to play with a lot," a second contained a neutral face for "children you kinda like to play with," and the third contained a sad face for "children you don't like to play with." Prior training with pictures of different foods established that each child understood the rating procedure. Ratings were assigned a score of 3 for positive, 2 for neutral, and 1 for negative, in order to obtain a composite score in the form of an overall rating. In addition, separate scores were obtained for the number of positive assignments and the number of negative assignments.

Reliability. Prior to coding, five raters were trained for a period of 12–19 weeks on the two observation scales. Videotapes of pilot play groups were used for training and final prestudy reliability assessments. Following the training program, all raters achieved the minimum average criterion necessary for participation of 70% interobserver agreement for each of the major categories for ten 10-min segments from a reliability tape (containing complex segments) for each of the two scales. Reliability also was obtained for each rater during the course of the study for 25% of the play group tapes selected on a random basis but balanced to ensure representation from the two types of social settings, groups, and time.

For the social participation and cognitive play scale, reliability was based on percent agreement obtained across each of the 10-s observation intervals (number of agreements divided by the total number of observations and transformed to a percentage). Cohen's (1960) kappa also was calculated where appropriate. For prestudy reliability, raters agreed on a mean of 84% (range 83–85%) of the intervals ($\kappa = .80$) for the 10 categories of the social participation scale. Using only those instances in which observers agreed that a cognitive play coding was required, interobserver agreement averaged 94% (range 93–96%) for the four cognitive play categories. Average agreement with regard to the identity of the peer involved in the social interaction was 85% (range 80–93%). During the course of the study, average interobserver agreement continued to be high in all instances for each of the 12 groups: social participation, 86% (range 82–90%), $\kappa = .81$ (range .76–.85); cognitive play, 91% (range 82–97%); and the identity of the peer, 90% (range 84–96%).

For the individual social behavior scale, raters were considered to be in agreement if codes matched within a specified 10-s interval using the "best fit" matching method (Hollenbeck, 1978). A reliability manual describing this method is available from the first author. In addition to the 34 individual social behavior categories, a "no-interaction" event was included to complete the possible options within each interval. Percent agreement was obtained for each 10-min segment by taking the total number of agreements, dividing by the total number of observed individual social interactions, and transforming to a percentage. Calculated in this manner, the average prestudy agreement for this scale was 85% (range 84–87%), $\kappa = .75$. Given agreement on the occurrence of a particular social interaction, observers further agreed on an average of 82% (range 80–90%) of the occasions as to whether the event could be classified as successful or unsuccessful, an average of 79% (range 67–88%) as to whether or not selected focal child behaviors were initiations, and an average of 98% (range 97–99%) as to the identity of the peer involved in the social interaction. Mean reliabilities for observations carried out during the course of the study (25% of the total) were as follows: individual social behaviors, 87% (range 83–92%), $\kappa =$ .78 (range .76–.83); successful/unsuccessful, 91% (range 84–100%); initiations, 80% (range 67–96%); and identity of peer, 96% (range 91–99%).

Results

Overview

We will begin the results section by discussing the differences in the model parameters for the three different kinds of children: developmentally delayed, communication disordered, and normally developing. We will then examine specific contrasts for model parameters comparing the mainstreamed with the specialized setting for developmentally delayed and communication disordered children. We will then present an analysis of the validity of the mathematical model parameters, with respect to the small set of criterion variables we have selected.

Differences in selected model parameters across types of children: What does the model tell us?

Inertia parameters. The inertia parameter, r, is an index of the steadiness of the relative positivity to negativity of the focal child's interaction. For the focal child's inertia parameter, $F(2, 123) = 2.98$, $p = .0543$, with means: developmentally delayed = −.04, communication disordered = .04, and normally developing = .11. Subsequent tests (using the Multiple Range Test, LSD procedure) revealed only that a normally developing focal child had more emotional inertia than the developmentally delayed focal child. Our first result is that what developmentally delayed children bring to the group ecology is a *temporally more erratic and staccato affective pattern* than the interaction of the normally developing child. For the *group*'s inertia parameter, F $(2, 97) = 5.10$, $p = .0079$, with means: group with developmentally delayed = −.30, group with communication disordered = .47, and group with normally developing = .07. Subsequent tests showed that the inertia of the group interacting with a communication disordered focal child was greater than the group's inertia when interacting with the developmentally delayed child. Thus, the group's inertia when interacting with the communication disordered child was greater than the group's inertia when interacting with the normally developing child but that the group's inertia when interacting with the developmentally delayed child was not significantly different from that of the group when interacting with the normally developing child. Hence, unlike what the developmentally delayed child brought to the group, what *the group* brought to the interaction when the group interacted with the communication disordered focal child was *greater* inertia than when interacting with either the normally developing or the developmentally delayed focal child. These data support our concept that emotion regulation is, in part, in the social ecology of peer interaction.

The a and the b parameters. The a and the b parameters represent the relative amount of positivity the child or the group, respectively, initially bring to the interaction, independent of social influence. For the focal child's parameter, a, there were no significant differences across groups, with F $(2, 123) = .28$, n.s., with means: developmentally delayed = 1.07, communication disordered = 1.19, and normally developing = 1.01. However, for the group, and the parameter, b, F $(2, 123) = 5.62$, $p = .0046$, with means: group with developmentally delayed = 1.42, group with communication disordered = −.79, and group with normally developing = .58. Subsequent tests revealed that what the group brought to its initial interaction with the developmentally delayed children was more positive than for the communication disordered, what the group initially brought to interaction with the normally developing child was greater than for the communication disordered, and what the group brought to initial interaction with the developmentally delayed was equal to what the group brought to interaction with the normally developing child. Hence, despite the fact that the focal child does not bring a more positive or a more negative initial state to the group, groups interacting with either a normally developing child or with a developmentally delayed child are more positive than when the group interacts with a communication disordered child. Once again, we see that, in addition to the steadiness of the emotional balance of positivity and negativity, the group ecology is providing a more positive initial start point for the delayed and normally developing child.

Effects of the mainstreamed compared to the specialized ecology: Influenced minus uninfluenced set points

Perhaps the most important question about the emotional regulatory effects of the mainstreamed setting is whether the mainstreamed group's ecology moves either itself or the focal child to a more positive set point than either the focal child or the group brings to the interaction before social influence, when compared to the specialized group ecology. For the developmentally delayed child, the differential effect of the two ecologies on the focal child was nonsignificant, F $(1, 51) = .09$; both mean differences were near zero: .02 for the specialized ecology and −.03 for the mainstreamed ecology. However, the effect of the focal child *on the group* was significantly different across the two ecologies, F $(1, 51) = 4.54$, $p < .05$. The effect of the focal child on the group in the specialized group was to move the group toward a more negative set point (mean change = −.38), whereas the effect of the focal child on the group in the mainstreamed group was to move the group toward a more positive set point (mean = .23). There were no significant effects of the mainstreamed compared to the specialized setting for communication handicapped children, for either the focal child or the group.

Effects of the mainstreamed compared to the specialized ecology: Comparison of selected model parameters

Comparison of specific model parameters across ecologies. Only the two specific contrasts between specialized and mainstreamed setting for developmentally delayed and for communication disordered were performed. The effect will be referred to as a "setting" effect. The t tests used for the contrasts employed the mean square error term from the one way (5-group type) univariate analyses of variance, with no experimentwise protection alpha.

Inertia. For developmentally delayed children the focal child's inertia parameter was significantly higher in the mainstreamed setting, .04 versus −.08, $t(34) = 2.40$, $p < .05$. This means that in the mainstreamed setting the developmentally delayed child's interaction is less erratic and less staccato, that is, more like the normally developing child's inertia. This suggests that the mainstreamed ecology provides a great function of emotion regulation, smoothing the influenced affective set point of the developmentally delayed child. The differences were not significant for communication disordered children, $t(34) = 1.40$, n.s. For developmentally delayed children and for communication disordered children there were no significant effects of setting on the group's inertia parameter when interacting with the focal child.

Positivity of what the focal child and the group initially brings to the interaction. There were no significant setting effects for the focal child's uninfluenced set point. However, for developmentally delayed children there was a significant effect of what the group brought to the interaction as a function of setting, $t(34) = 2.09$, $p < .05$, with the uninfluenced start value higher for the specialized group (.14), compared to the mainstreamed setting (.07). This finding is likely to be offset by the dramatically lowered amount of interaction for developmentally delayed children in the specialized setting (Guralnick, Connor, Hammond, Gottman, & Kinnish, 1996a, 1996b) (see Appendix C).

Summary plot of the influence functions. The influence functions provide the *dynamics* of how the group and the focal child affect one another and create changes in one another over time. Figure 5 contains four plots: Figure 5a is a plot of the child's influence on the group for normally developing children in specialized groups compared to developmentally delayed children in specialized and in mainstreamed groups; Figure 5b is a plot of the group's influence on the focal child for normally developing children in specialized groups compared to developmentally delayed children in specialized and in mainstreamed groups. Figure 5c is a plot of the child's influence on the group for normally developing children in specialized groups compared to communication disordered children in specialized and in mainstreamed groups; Figure 5d is a plot of the group's influence on the focal child for normally developing children in specialized groups compared to communication disordered children in specialized and in mainstreamed groups.

First, let us consider the developmentally delayed child in both settings, compared to normally developing children in their specialized groups. Examine the influence of the focal child on the group. We can summarize these plots by concluding that *for developmentally delayed children, the mainstreamed setting has changed the effects of the developmentally delayed child on the group so it is closer to that of the normally developing child.* Let us next examine the influence of the group on the focal child. Consider first developmentally delayed children in both settings, compared to normally developing children in their specialized groups. Examine the positive range of Figure 5b. For developmentally delayed children, in the positive range of the figure, we see our finding that there is a

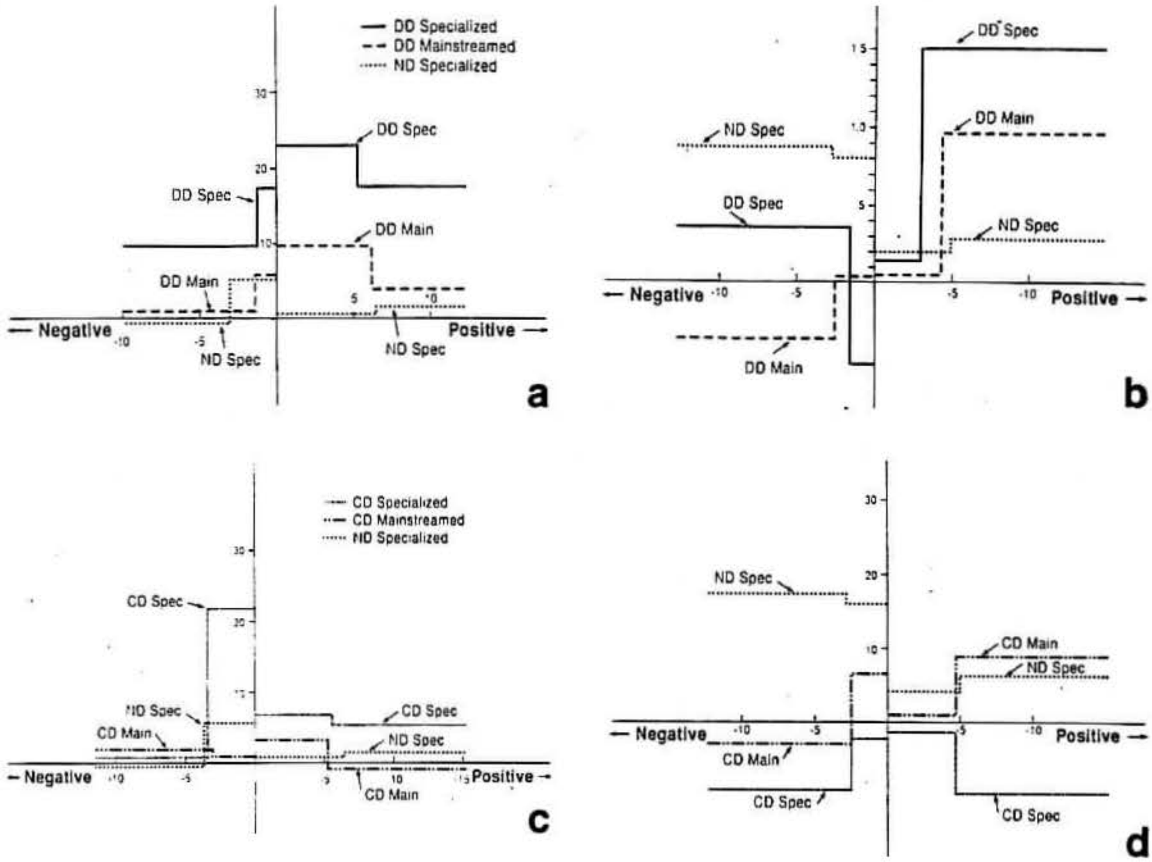


Figure 5. Influence functions for developmentally delayed and communication disordered compared to normally developing children in specialized groups: (a) focal child's influence on the group, developmentally delayed children; (b) group's influence on the focal child, developmentally delayed children; (c) focal child's influence on the group, communication disordered children; (d) group's influence on the focal child, communication disordered children.

lower threshold of positivity before the group has a positive impact on the focal child; this is true in both the specialized and the mainstreamed setting. Thus, we see again that the mainstreamed setting has changed the effects of the group on the developmentally delayed child in the high positive range so it is closer to that of the normally developing child. In the negative ranges, we can see that in the mainstreamed setting, the group's negativity had a more negative influence on the developmentally delayed child than was the case in the specialized setting.

Second, let us consider the communication disordered child. First, examine the influence of the focal child on the group. Let us first consider communication disordered children in both settings, compared to normally developing children in their specialized groups. Examine the positive range of Figure 5c. For communication disordered children, we see that the mainstreamed setting has also changed the effects of the child on the group so it is closer to that of the normally developing child. In the negative ranges, we can see this same effect: in the mainstreamed setting, the communication disordered child's negativity had a more negative influence on the group than was the case in the specialized setting. Now examine the influence of the group on the focal child. Let us first consider communication disordered children in both settings, compared to normally developing children in their specialized groups. If we examine the positive range of Figure 5d, we see that the mainstreamed setting has again changed the effects of the group on the communication disordered child in the positive range so it is closer to that of the normally developing child.

However, in the negative ranges, we see that in the mainstreamed (compared to the specialized) setting, the group's negativity had a more positive influence on the communication disordered child than was the case in the specialized setting; this pattern is opposite to the effect of mainstreaming for developmentally delayed children. Hence, in the mainstreamed (compared to the specialized) setting the developmentally delayed child had a greater sensitivity to the group's negativity, while the communication disordered child had less sensitivity to the group's negativity.

Strength of the attractors

In our 3-dimensional plots we assess the size of the peaks in the negative–negative quadrant with two measures, the average elevation and the maximum elevation of the "mountain range" in this quadrant. Recall that this elevation is an index of the group ecology being able to buffer the focal child away from high levels of negativity. It is an index of strength of *the ecology's emotional regulation* in interaction with the focal child. A higher mountain peak means that, if perturbed away from a set point, there will be more rapid return to the set point; thus, the size of the mountain peak is an index of the strength of attraction, or the amount of stability of the set point. Our two measures of this construct correlated .98, $p < .001$, across the subjects in this study, so we examined only the maximum elevation, which we called "Z_{max}." In examining the validity of Z_{max}, we employed the Rubin POS and found that Z_{max} was significantly correlated with adult intervention (usually to diminish conflict) (.24, $p < .01$), group functional play (.23, $p < .01$), and parallel functional play (.26, $p < .01$). We correlated Z_{max} with five factors of the ISB: (a) social engagement, .21, $p < .01$; (b) inclusion processes (includes imitates/agrees), .12, ns; (c) influence processes (includes follows peer), which includes being influenced, refusing to accept influence, and influencing, .24, $p < .01$; (d) responsiveness to social cues, .10, ns; and (e) aggression, .49, $p < .001$. This is a picture in which we see that the buffer variable Z_{max} is understandably related significantly to the child's engagement in processes involving influence and conflict. Using a one-tailed t-test we found that normally developing children had significantly lower Z_{max} than developmentally delayed children, $t(93) = 1.92$, $p < .05$ (developmentally delayed mean = 163.83, normally developing mean = 112.99). Hence, *the group ecology is doing significantly more emotion regulation or buffering for the developmentally delayed child than it is for the normally developing child.*

Validity of the mathematical model parameters

Table 1 is a summary of the correlations between the parameters of the mathematical model and the criterion variables derived from the Rubin Play Observation Scale, the peer sociometric measures, and the two Guralnick indices, reciprocal friends and the number of children who choose the focal child as a play partner. These relationships are summarized in Table 1. As Table 1 indicates, with respect to these criterion variables, we need only employ a few model parameters to account for significant variation in the criterion variables. For example, although the inertia parameters had low validity, with only one correlation significant out of 12, the uninfluenced start values, however, were related to the criterion variables. These index the relative positivity that the focal child and the group bring to the interaction independent of social influence processes. The more positive the start values, particularly indexing the amount of relative positivity the focal child brings to the interaction, the more likely the child is to engage in conversation, to receive positive sociometric nominations, to have reciprocal friends, and to be sought out as a play partner by other children in the group. The positive start values of the group toward the focal child are related to the probability that the focal child will engage in group dramatic play, and negatively related to negative sociometric ratings.

Higher positive thresholds were related to the criterion variable, both for the focal child's influence function and for the group's. This means that to the extent that a higher threshold existed for change in positivity re-

Table 1. *Validity of parameters of mathematical model*

Parameter	Dramatic Play	Conversation	Positive Sociom.	Negative Sociom.	Reciprocal Friends	Children Seek Out Child
	Criterion Variable					
Inertia						
r_1	.09	.05	−.10	−.04	−.06	−.11
r_2	−.20*	−.06	.15	.07	.04	.02
Start (uninfluenced model constant)						
a	.10	.23**	.30***	−.09	.20*	.30***
b	.15*	.03	.02	−.15*	−.03	−.04
Uninfluenced set points						
$a/(1-r_1)$	.10	.10	.12	−.03	.10	.14
$b/(1-r_2)$	.09	.02	.19*	−.10	.04	.09
Influenced set points						
Peer	.15*	.13	.12	−.06	.11	.18*
Focal child	.07	.20*	.16*	.03	.11	.18*
Thresholds						
Positive						
C_s	.14	.35***	.23**	−.16*	.21**	.33***
C_g	.24**	.33***	.29***	−.31***	.12	.26**
Negative						
F_s	−.14	−.21**	.02	−.04	−.15*	−.10
F_g	−.04	−.07	.09	−.12	.14	.13
Influence function parameters						
Positive						
A_f	−.18*	−.20*	−.21*	.06	−.15	−.16*
A_g	−.02	.00	−.04	.10	−.04	.04
B_f	−.15	−.30***	−.15	.07	−.14	−.15
B_g	−.02	.05	.11	−.10	−.01	.10
Negative						
D_s	−.18**	−.33***	−.21*	.23**	−.09	−.06
D_g	.01	−.01	−.11	.02	−.07	−.09
E_s	−.08	−.01	.03	−.16*	−.03	−.10
E_g	.06	−.10	−.12	.12	.00	−.10

Sociom., sociometrics.
*$p < .05$. **$p < .01$. ***$p < .001$.

sulting in a change in the partner, the criterion variables were higher. The criterion variables were also significantly related to the influenced set points, particularly those of the target child.

Discussion

In our field we have implicitly assumed that the locus of emotion regulatory competence is primarily within the developing child. The purpose of this paper was to examine if it would be useful to attempt to change the locus of the concept of emotion regulation from the child to the interaction of the child with a particular peer social ecology. The goal of this reconceptualization was to create a model of emotion regulation that would take us from the individual child's physiology to the child's interactive social ecologies.

We suggested that it would be useful in this regard to introduce a new approach using mathematical modeling based upon a set point theory of the regulatory balance between positive and negative affect. The nonlinear mathematical model we proposed was able to separate the emotional regulatory set points that the focal child initially brings to a group, or that the group initially brings to the focal child, from the social influence functions that represent the mutual influences resulting from interaction, and these functions, together with the uninfluenced set points determine what we called the "influenced set points." Thus, we can think of emotion regulation in two parts: emotion regulation within the child (the parameters that describe the uninfluenced portions of what the child brings to the group) and emotion regulation within the ecology (the parameters that describe the interaction of the child with the group).

One question we can immediately ask is whether, as a function of social ecology, the influenced set points are more positive than the uninfluenced. We note that we could not even ask this question without the mathematical model, which separates emotion regulation into its two parts, emotion regulation within the child and emotion regulation within the ecology.

We are greatly encouraged by the success we had with these data in describing what the various types of children we studied and what the differential ecologies initially bring to the interaction, and also in our ability to describe how these ecologies influence and are influenced, in terms of the emotion regulatory steady states. It was our contention that the mathematical modeling we proposed would add some theoretical clarity and descriptive power to standard notions of emotion regulation. The same mathematical model has already added great clarity to our work with marriages.

Guralnick et al. (1996a, 1996b), using traditional statistical analyses, found results that were quite consistent with the results of the analyses presented here. They reported that developmentally delayed children were unoccupied twice as often in the specialized than in the mainstreamed setting and engaged in more parallel play in the mainstreamed than in the specialized setting. There were also major effects of setting on children's socially interactive behavior both as revealed by the White/Watts Scale and in a level of social interaction factor emerging from a principal components factor analysis (see Guralnick et al., 1996a discussion, pp. 373–374, 1996b). Interestingly, the benefits of type of ecological setting in that study were found for both the delayed and nondelayed groups.

In addition to being consistent with other analyses, the mathematical modeling added something new. To illustrate the kind of description that the mathematical modeling made possible, consider the dynamic nature of our results for the developmentally delayed child. The mathematical model presented the results in a simple theoretical language that provided a *systemic* notion of emotion regulation, one that involved set point theory, in which a particular behavioral affective set point or homeostatic setting was defended by the interacting system. In the theoretical language we proposed, the focal child would be continually attracted to its natural *uninfluenced* set point, except that nonlinear interactions within the interacting system change the child's trajectory so that it is instead drawn to its *influenced* set point. The equations of the system we proposed included the form of these nonlinear influence functions. Since they are empirically fit, they are an extra byproduct of our modeling. Thus, we would contend that, at the outset, the mathematical model provides a new dynamic language for describing the forces that shape a child and a group's interaction over time.

In summarizing our analyses using the mathematical modeling, we start with what the focal child initially brings to the group and what the group initially brings to the focal child. We found that the developmentally delayed focal child does not bring a more negative uninfluenced initial state to the group than other children. However, we also found that for developmentally delayed children there was a significant effect of what the group brought to the interaction as a function of the ecological setting. The mainstreamed group brought a far more positive uninfluenced set point to its interaction with the developmentally delayed child than it would have otherwise.

We developed the mathematical model to represent our set point theory because it is our eventual goal to integrate interactive behavioral physiological data in the study of developmentally delayed children's peer interactions. The child's body is also designed to maintain a number of dynamic equilibria, homeostases, or set points. The analogue is that the body's response to the environmental demands facing the developing child requires the child's neural control systems to maintain a flexible, adaptive homeostasis. The child's ability to self-regulate around these set points forms the basis of any adaptive response.

For example, consider a centerpiece of our theory of the role of emotion regulation in the development of social competence in the developmentally delayed child. We predict that *positive affect* should play a much greater role for developmentally delayed children than it does for normally developing children. What do we expect would be the effects on developmentally delayed children of the more positive uninfluenced peer emotional climate in the mainstreamed ecological context? Based upon our pilot work and a study in progress with the families of developmentally delayed children, we believe that our finding of the increased positive affect of the mainstreamed group toward the developmentally delayed child is absolutely critical for the development of basic regulatory capabilities in the delayed child that involve *attention and its underlying regulatory physiology.* We are finding that, independent of measured IQ, those developmentally delayed children who are the most socially competent in the peer entry situation are those children (a) whose parents have low levels of marital conflict, (b) high spousal warmth and cooperation, (c) low interparent competition in the Cowan Make-A-World task, and in general had (d) siblings and parents who were affectively warm. These results appear to be quite dramatic. Based on our pilot research in progress we suggest that there are two physiologically based regulatory processes that are greatly affected by social interactions that start off positively. We suggest that these regulatory processes are greatly affected by those emotional processes that engage the family's "affectional system."

It appears that attentional processes mediate between risk and the development of a range of psychopathologies in children (Kellam et al., 1991). We suggest that attentional capabilities in children are a sensitive lead indicator of the development of psychopathology because these abilities are formed by the nature of affective interaction in families. Furthermore, these capabilities appear to have their underpinnings in the physiological bases of the ability to self-soothe and the physiological bases of the ability to engage with the world in effortful information processing.

The two physiological components of emotion regulation are *vagal tone* and *the ability to suppress vagal tone* (for a review of these physiological constructs, see Wilson & Gottman, 1996, and Gottman & Wilson, 1997). These constructs are inspired by the work of Porges and his associates (e.g., see Porges et al., 1994). We propose that these two physiological variables are, in turn, the underpinnings of a two-component model of attention, *the ability to sustain attention* and *the ability to shift attention.* We suggest that these abilities are part of a nexus of self-regulatory skills that are related to emotion regulation, which, in turn, can form the basis of peer social competence.

What might be the mechanism of this linkage between positive affect, physiology and attention? *We propose that the increased affection and positivity serves the role of soothing the child and helping the child focus attention.* These external scaffolds for self-soothing and focusing attention are eventually internalized, but, at first, the location of emotion regulation ability is in the interaction of the delayed child with the mainstreamed peer group. Paradoxically, we suggest that the developmentally delayed child both needs more of this positive-affect scaffolding than the normally developing child, and that the response of the delayed child is greater. The mainstreamed interactive context, then, may provide the regulatory scaffolding necessary for the developmentally delayed child to function interactively much more like a normally developing child.

This thinking is consistent with the results we see in the other parameters of the mathematical model. For example, consider the inertia parameter of our model. First of all, note that the mathematical modeling makes it possible for us to discuss a complex patterned response like "inertia." Our findings with the inertia parameter suggest that, compared to the normally developing child, the developmentally delayed child's peer group interaction is more erratic and staccato, but that this difference is reduced significantly in the mainstreamed compared to the specialized setting. These results speak well to the potential scaffolding benefits of the mainstreamed compared to the specialized setting for developmentally delayed children.

Thus, we can see that an advantage of the mathematical modeling we have proposed is that it makes it possible to tease apart the uninfluenced and influenced components of interaction, so that we may generate testable hypotheses about the effects of *context* on the interactive parameters of the developmentally delayed child's transactions with the peer group.

There are implications of our results for subsequent research that attempts to integrate the child's physiology and the family's affective context with the peer context. We expect that the inertia parameters will be significantly related to the child's emotional responses and the child's physiological reactions to emotion, particularly the kind of emotion dynamics that Thompson has delineated so carefully. It is well known that high levels of autonomic arousal make it more likely that people will perseverate with overlearned behaviors and cognitions, perhaps with those that are temperamentally central to the child (e.g., see Eysenck, 1982). The fact that the developmentally delayed child's inertia increases significantly in the mainstreamed setting, approaching that of normally developing children, suggests that this setting may be more autonomically arousing for the developmentally delayed child. We also know that it is initially (group uninfluenced set point) more positive. Hence, we would predict that the mainstreamed peer setting would interact with the family's affective climate. Those developmentally delayed children reared in families that are more positively affective should show higher inertia in the mainstreamed peer context.

We discovered several other things using the approach that permits us to locate part of "emotional regulation" in the child's interaction with a particular peer social ecology. In addition to the effect of the mainstreamed setting in reducing affective staccato of the developmentally delayed child, it also changed the group *influence patterns* for all specialized children so that they were closer to those in normally developing peer ecological groups. Furthermore, these effects were a two-way street. The focal child was influencing the group to a more positive set point in its interaction that it would have been drawn to without his influence. That is, the data suggest that these effects in the shift of the affective set point are effects that the focal child has on the group's ecology, rather than the other way around. These results are also consistent with Guralnick et al. (1996a, 1996b).

The approach we have outlined in this paper may be helpful in research on psychopathologies in which the regulation of affect within an interpersonal context is a central concern, such as depression, bipolar affective disorder, or borderline disorder. Observation of dyadic (or group) interaction in salient relationships (e.g., friendships, parent–child, or marital relationships) can be used to set up the basic time two dyadic series of positive-minus-negative affect, and then the parameters of the model and the influence functions can be computed for each target pathology, compared to a suitable control group. The suitable control group ought to be a normal control, or a comparable group in terms of demographics and exposure to risk variables. For the depressed group, we would predict that the uninfluenced set point would be considerably more negative than the normal control, and that the inertia parameter, which indexes the rapidity with which the individual returned to a depressed negative state, when not influenced, would be close to unity. For the bipolar group, we would have to modify the equations; for example, it would be adequate to include a *delay parameter* in the influence function (the one influencing the identified patient) so that affective behavior might be characterized by oscillation of suitable period. For the borderline personality disorder, their affective lability could be modeled by an inertia parameter close to zero, compared with a normal control group. It would be of great interest to see empirically what the form of the influence functions would be. In all pathologies, one could initially hypothesize that the influenced set point would be considerably more negative than the uninfluenced set point, compared to a normal control group.

To summarize, in our current context of the social interaction of preschool boys, in addition to the uninfluenced start point of the group being more positive in the mainstreamed context, the effects on the mainstreamed setting of the developmentally delayed child was to create an influenced set point that increases the positivity of the stable steady state of the group. The effects of the group are to change the influence patterns toward that of the normally developing peer ecology and to move the affective tempo of the developmentally delayed child to a more stable state. These are quite remarkable results and ones that could only be teased apart clearly by using the mathematical model, and the theoretical language of set point theory.

We found fewer significant effect for communication-disordered children. In some senses, things are harder in groups for the communication disordered child than for the developmentally delayed child. For example, for communication disordered children our results suggest that the group's initial level of positivity is higher for the developmentally delayed child than for communication disordered, but for the developmentally delayed child it was equal to that of the normally developing child.

This paper has been the first application of nonlinear mathematical modeling using difference equations to the study of interactive processes in children's groups. We believe that this paper demonstrates the potential usefulness of this approach. We also suggest that this conception, which involved emotional balances within a particular peer social ecology, will be well suited to attempting an integration of social with physiological conceptions of regulation. We expect that some of the parameters of our model will be related to the child's baseline physiology, as well as the child's physiological reactivity and the speed of the child's recovery from arousal.

There are unexplored aspects of the model we proposed. We briefly mention the study of the differential thresholds of our influence functions, and the potential the model has for representing potential positive or negative "catastrophes." Catastrophes in mathematics are sudden dramatic *qualitative* changes that can occur in a system, as a model parameter changes gradually and slowly, until it crosses a critical threshold; they can be positive or negative in nature. This idea is like the old "straw that broke the camel's back" idea; the addition of just one more straw was enough to change the camel forever. In our model of the children's groups it is possible to change the parameters so that the system loses all its positive set points, or, conversely, to change the parameters so that the system loses its negative set point. In these cases the model would predict that the group's behavior would change dramatically. The model is capable of representing these very dramatic possibilities.

We briefly discuss the implications of this modeling for subsequent research. It is our contention that it would be profitable to study the balance of positive-to-negative affect in the families of developmentally delayed children, teasing apart what family members bring to salient social interactions from how people are influenced by these systems, studying both contexts that pull for conflict resolution and contexts that pull for positive affect. MacDonald and Parke (1984) found that positive affect contexts, particularly in father–child interactions were most predictive of the child's peer popularity. They proposed that the positive affect play systems that fathers engage so well are designed to teach the child to calm down affectively and physiologically from high levels of excitement.

Specifically, we propose that those families who are less conflicted, less competitive, and more affectionate will have developmentally delayed children who are better at focusing attention and who have higher basal vagal tone. Basal vagal tone is related to the ability to inhibit inappropriate behavior and act less impulsively (for a review see Wilson & Gottman, 1996). Furthermore, we suggest that these same children will also be better at engaging with the world when it requires shifting attention, the physiological dimension related to the ability to suppress vagal tone; suppressing vagal tone and increasing the heart's output is part of one's ability to engage in a task when the world poses cognitive or social demands. We propose that both abilities, focusing/sustaining attention and attention shifting, are essential for peer social competence. Wilson (1994) found that the developmentally delayed child's ability to attend and to shift attentional focus across changing affective instructional sets was significantly correlated with a measure of social attention during peer group entry. The tasks were to scan for and touch astronauts when they appeared in a passing spaceship on the video screen, and then to shift, scan for, and touch the video screen when a happy astronaut was in a passing spaceship, and then to shift to only touching the screen when an angry astronaut was in the passing spaceship. Hence, we propose that these two attentional abilities will be strongly related to concomitant basal vagal tone and the ability to suppress vagal tone (for measurement of the latter construct, see Gottman & Wilson, 1997). We expect positivity in the peer system to be equally important in shaping emotion regulatory abilities in the developing delayed child. The family, we expect, lays the groundwork for these abilities, and the abilities become amplified by positive peer contexts.

To summarize, in our theory, positive affect (in the family and in the peer system) plays a key role among developmentally delayed children in providing soothing experiences that form the basis of the child's regulatory physiology. This regulatory physiology is integrated with processes of behavioral inhibition, focusing attention, and attention shifting in response to environmental demands of the ecology. These, in turn, form the basis of the child's social competence in that ecology.

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Appendix A

S refers to the focal child (subject), and P refers to the peer the child is interacting with (group):

1. involved observation $S = 0$, $P = 0$
2. joins peer in specific activity $S = +2$, $P = 0$, if S initiated $S = +3$
3. verbally supports peer's statement $S = +2$, $P = 0$, if S initiated $S = +3$
4. verbal competition $S = -2$, $P = 0$, if S initiated $S = -3$
5. shows pride $S = +1$, $P = 0$, if S initiated $S = +3$
6. competes with peers for adult attention $S = -1$, $P = 0$, if S initiated $S = -2$
7. expresses affection to peers $S = +3$, $P = 0$, if S initiated $S = +4$
8. shows empathy $S = +4$, $P = 0$, if S initiated $S = +5$
9. expresses hostility to peers $S = -6$, $P = 0$, if S initiated $S = -7$

Next codes, depends on double coding as successful or unsuccessful:

10. lead in peer direct positive, $S = +2$, (+3 if initiated), $P = 1$, (2, if S-initiated). If successful, add +1 to S and P
11. lead in peer indirect positive, $S = 3, 4$; $P = 1, 2$, if S-initiated. If successful add +1 to S and P
12. lead in peer direct negative, $S = 2, 3$; $P = 0, 2$, if S-initiated. If successful add +1 to S and P
13. lead in peer activities, indirect negative, $S = 3, 4$; $P = 3, 4$, if S initiated. If successful add +1 to S and P

Next codes if there is no successful or unsuccessful double code:

20. follows lead of peer direct positive or neutral, $S = 1$, $P = 3$
21. follows lead of peer indirect positive or neutral, $S = 2$, $P = 3$
22. follows lead of peer, direct negative, $S = 3$, $P = 2$
23. follows lead of peer, indirect negative, $S = 4$, $P = 1$
30. refuses to follow lead of peer, direct positive, $S = 0$, $P = -1$
31. refuses to follow lead of peer, indirect positive, $S = 1$, $P = -2$
32. refuses to follow lead of peer, direct negative, $S = 2$, $P = -3$
33. refuses to follow lead of peer, indirect negative, $S = 2$, $P = -4$
40. peer as resource $S = 5$, $P = 5$, if S initiated, $S = 6$
41. responds to peer's use of S as resource, $S = 3$, $P = 2$
42. fails to respond to peer's use of S as resource, $S = 3$, $P = 2$
50. takes unoffered object, $S = -5$, $P = 0$, if S initiated still leave as -5
51. defends property, $S = 2$, $P = -2$
60. imitation of peer, $S = 3$, $P = 1$
61. being a model, $S = 3$, $P = 1$
70. seeks attention of peer, $S = 3$, $P = 1$, if S initiated $S = 4$
71. responds to peer's attention-seeking behavior, $S = 3$, $P = 1$
72. fails to respond to peer's attention-seeking behavior, $S = -3$, $P = 1$
80. seeks agreement from peer, $S = 2$, $P = 0$, if S initiated $S = 3$
81. responds to P's efforts to seek agreement, $S = 4$, $P = 3$
82. fails to respond to P's efforts to seek agreement, $S = -4$, $P = 3$

Appendix B

We also analyzed specific model parameters, and we present selected results here.

Thresholds of positivity

The threshold of positivity parameter is denoted *C*. For the focal child, the positive threshold parameter, F (2, 123) = 6.55, p = .0020, with means: developmentally delayed = 3.33, communication disordered = 4.40, and normally developing = 4.98. Subsequent tests revealed that the focal child's positive threshold was greater for both communication disordered children and normally developing children than for developmentally delayed children. This means that, compared to the other two types of children, the developmentally delayed focal child had a significantly lower threshold of positivity before the child's effect on the group became more positive. That is, one could conclude that *the developmentally delayed child had to do less, that is, he can be less positive before there is an increased positive impact on the group*, compared to when the focal child is a member of one of the other two groups of children. In a sense, then, one can say that the group is more easily pleased with lower levels of positivity from the developmentally delayed child. This result is independent of what the child brought to the group. It is entirely an effect of the group ecology.

Effects of low levels of positivity

For the focal child, the A parameter of the influence function assesses the effect on the group of the lower amounts of positivity of the focal child. For this parameter, F (2, 123) = 5.00, p = .0084, with means: developmentally delayed = 19.27, communication disordered = 4.29, and normally developing = 4.07. This means that *lower levels of focal child positivity had much greater positive effects on the group when the focal child was developmentally delayed than when the focal child was either communication disordered or normally developing.*

For the group, and the A parameter, F (2, 123) = 4.26, p = .0126. The means were: group with developmentally delayed = .10, group with communication disordered = −.09, and group with normally developing = .16. Subsequent tests revealed that the group interacting with the normally developing child was more positive than with the communication disordered child, but that the group interacting with the normally developing child equaled the group interacting with the developmentally delayed child. Lower levels of group positivity for both normally developing and developmentally delayed children had a greater positive impact on the focal child than equivalent levels of positivity for the group's interaction with communication disordered focal children.

Appendix C

Positive threshold

There was a significantly higher threshold for positivity in the mainstreamed compared to the specialized setting for the group interacting with developmentally delayed children (4.47 compared to 3.11, respectively), $t(34) = 2.72$, $p < .05$. This, in effect, means that to have a greater positive impact on the child, the group interacting with the developmentally delayed child has to be more positive in the mainstreamed than in the specialized setting. In effect, this suggest the interesting hypothesis that the developmentally delayed child may have higher expectations of positivity for other children in the mainstreamed than in the specialized setting. The setting effect for communication disordered children was not significant.

Negative threshold

There was a significantly higher threshold for negativity in the mainstreamed compared to the specialized setting for developmentally delayed children (−1.78 compared to −2.56), $t(34) = 2.60$, $p < .05$. This means that it takes less negative group behavior to have an increased effect on the child in the specialized compared to the mainstreamed setting. Since this effect is generally negative (see Figure 5), this suggests the interesting finding that the developmentally delayed child is more sensitive to negativity in the mainstreamed setting than he is in the specialized setting. The effect for communication disordered children was not significant. Thus, despite the buffering that this setting holds for the developmentally delayed child, it is also a somewhat higher risk setting. It is clearly an interactive setting with far more power to affect the emotion regulation of the developmentally delayed focal child.

Effects of low positivity

For developmentally delayed children there was a significant effect of setting, with $t(34) = 2.58$, with specialized = 22.77 and mainstreamed = 9.89, which suggests the interesting result that the mainstreamed developmentally delayed child has less influence on the group when he is low-level positive than he does when he is in the specialized setting. There were no setting effects for communication disordered children.

Effects of high positivity

For the focal child, $t(34) = 3.78$, with specialized mean = 17.62 and mainstreamed mean = 4.01. This implies that the developmentally delayed focal child is having less influence on the group when he is highly positive in the mainstreamed setting than he does in the specialized group setting. There were no setting effects for communication disordered children.

Effects of low negativity

For the focal child, there were no significant effects for the developmentally delayed child, $t(34) = 1.76$, but for communication disordered children, $t(34) = 3.37$, with specialized mean = 22.01, and mainstreamed mean = .37. Communication disordered children are having a more positive impact on the group with low negativity in the specialized setting than in the mainstreamed setting.

For the group's interaction with the focal child, for developmentally delayed children, $t(34) = 2.10$, $p < .05$, with specialized mean = −.62 and mainstreamed mean = .01, so that the effects of the group's low level negativity on the developmentally delayed child are far more negative in the specialized setting than in the specialized setting. There were no significant effect of setting for communication disordered children.

Effects of high negativity

For the group, there were no significant effects of setting. For the focal child, there was a significant effect of setting only for the developmentally delayed child, $t(34) = 2.38$, with the specialized mean = 9.86 and the mainstreamed mean = 1.11. Hence, for developmentally delayed children, high levels of negativity had a more positive impact on the group in the specialized setting than in the mainstreamed setting. This suggests a potential buffer created by the mainstreamed setting on higher levels of negativity by the developmentally delayed child. There were no effects of setting for the communication disordered child.